A Passivity Based Cartesian Impedance Controller for Flexible Joint Robots - Part I: Torque Feedback and Gravity Compensation

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***Abstract*— In this paper a novel approach to the Cartesian Impedance Control problem for robots with flexible joints is presented. The proposed controller structure is based on simple physical considerations, which are motivating the extension of classical position feedback by an additional feedback of the joint torques. The torque feedback action can be interpreted as a scaling of the apparent motor inertia. Furthermore the problem of gravity compensation is addressed. Finally, it is shown that the closed loop system can be seen as a feedback interconnection of passive systems. Based on these passivity property a proof of asymptotic stability is presented.**

I. Introduction

Cartesian impedance control, as a particular approach to the compliant motion control problem, clearly can be seen as one of the core techniques of modern robot control. The basic goal of impedance control in the most general sense is to achieve a desired dynamical relationship between external forces and movements of the robot [6]. However in many robotic applications this dynamical behaviour is specified in terms of stiffness and damping matrices with respect to some Cartesian coordinates.

The classical approach to impedance control concentrates on robotic systems in which the joint elasticity can be neglected. A straight forward application of these techniques to a flexible joint robot therefore usually will not lead to a satisfactory performance[1]. In this paper an impedance control law is proposed which is designed for flexible joint robots. The desired impedance is assumed to be a second order mass-spring-damper system. Furthermore only the achievement of stiffness and damping is considered herein, while the inertial behaviour is left unchanged.

In case of a robot with rigid joints, such a stiffness and damping behaviour could in principle be implemented quite easily with a PD-like controller (formulated in the relevant coordinates). In [12] it was proven that a motor position based PD-controller leads to a stable closed loop system also in case of a robot with flexible joints. Furthermore in [4] a stability analysis for a hybrid position/force controller[2] was presented. However it has been shown that in practice only quite unsatisfactory results can be achieved with a restriction to purely motor position (and velocity) based feedback controllers (without additional noncollocated feedback) for the case of a flexible joint robot. In some works a controller structure based on a feedback of the joint torques as well as the link side positions was considered and it was shown that this can lead to better results (see e.g. [10]). This has also already been verified experimentally with the DLR-light-weight-robots [2]. From a theoretical point of view this approach usually is justified (for sufficiently high joint stiffnesses) by an approximate analysis based on the singular perturbation theory. The feedback of the joint torques is therein considered as the control action for an inner control loop, which receives its setpoint from an outer loop impedance controller.

In [1] a controller with a complete static state feedback (position and torque as well as their first derivatives) was introduced, for which (analogously to [12]) asymptotical stability was shown based on the passivity properties of the controller. Contrary to the classical PD-controller, the motor inertia and the joint stiffness are included in the same passive block as the state feedback controller such that an effective damping of the joint oscillations could be achieved.

In this paper a physical interpretation of the torque feedback is given. With this a stability analysis is given which is based on the passivity properties of the system. It is important to notice that the described controller itself is not passive due to the feedback of the joint torque, but it will be shown that the controlled motor dynamics in combination with the torque feedback are passive. Together with the passive (link side) rigid body dynamics the closed loop system can therefore be represented as a feedback interconnection of two passive systems. This passivity property is ensured for the Cartesian impedance controller as well as for a joint level impedance controller.

Furthermore in [1], [12] a gravity compensation term based on the desired configuration was used. In case of an impedance controller this is not appropriate, due to the big deviations from the desired configuration which may occur here (in case of a low desired stiffness). In this work a gravity compensation

[1] In terms of damping out the oscillations due to the flexiblity in the joint.
[2] for a flexible joint robot without gravitational effects

term will be designed which is based on the measurement of the motor position and is better suited for the use in context with impedance control. The problem of gravity compensation for flexible joint robots in case of impedance control was also addressed in a recent paper of Zollo et al. [15]. However, in contrary to our approach, the gravity compensation term in [15] led to additional constraints on the admissible Cartesian stiffness.
A similar, passivity based, controller structure in which the controller can also be implemented without a measurement of the joint velocities is given by the *IPC*[3] from [11]. It should be mentioned that the basic idea of the presented work was the result of considerations about how such a passivity based controller designed for rigid body robots could be best implemented for a robot with flexible joints.
The presented controller is also strongly related to the state feedback controller from [1]. This will be explained in more detail in the second part of the paper [3]. Therein it will also be shown how the presented controller can (analogously to [1]) be extented to a complete state feedback form without loosing the passivity properties.
It should also be mentioned that the presented approach clearly relies on the availability of the joint torques, which can be achieved either directly by measurement or indirectly by an additional measurement of the link side position. The DLR-light-weight-robots [5] (Fig. 1) are equipped with joint torque sensors in order to allow the achievement of fine manipulation and to enhance the performance when the robot is in interaction with the environment. Therefore they are ideally suited for the implementation of the presented controller. In the second part of the paper also some experimental results of the Cartesian impedance controller with the DLR-light-weight-robot-II are shown.
This paper is organized as follows: In Section II the design idea is described based on a simplified one-dimensional model. The generalization of the design idea to the complete model of a flexible joint robot is then presented in Section III. In Section IV the controller will then be augmented by an appropriate gravity compensation term. Finally, a detailed passivity and stability analysis is given in Section V.
The second part of this paper [3] will deal with some further extentions of the controller, which are important in practice. These extentions contain on the one hand more details about the actual design of the damping and stiffness matrices. On the other hand, also the generalization of the controller to a complete state feedback form is described therein.

II. Design Idea

In this section the basic idea underlying the controller design is described. It is motivated by some simple considerations for a simplified one-dimensional model.
Consider at first the model of a single flexible joint as it is sketched in figure 1 for the second joint of the DLR-light-weight-robot-III. The motor torque τ_m acts here on the rotor inertia B of the motor[4]. The elasticity of the transmission between the rotor and the following link of the robot[5] is modelled in form of a linear spring with stiffness K.

[3] *Intrinsically Passive Controller*

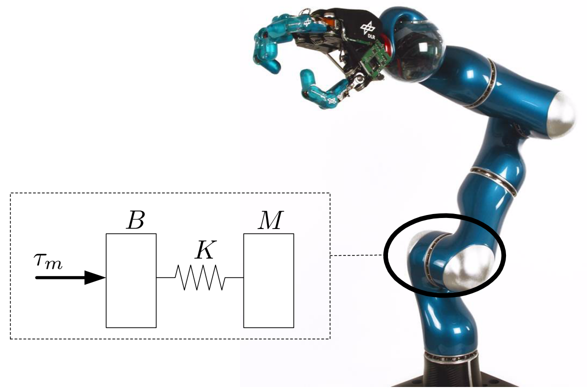

Fig. 1. Sketch of the Model for a Flexible Joint Robot.

The goal of the impedance controller is to achieve a desired dynamical behaviour with respect to external forces and torques which act on the link side. In the following it is assumed that this dynamical behaviour is given by a stiffness parameter K_θ as well as a damping parameter D_θ.
In case of a robot with rigid (i.e. non-elastic) joints this behaviour could be realized by a simple PD-controller. If one uses, as it is shown in figure 2 for the one-dimensional case, a motor position based PD-controller also in case of a robot with elastic joints, then the resulting dynamics will obviously be influenced significantly by the joint elasticity and the motor inertia. Intuitively speaking, the deviation from the desired behaviour will be less significant when the rotor mass B gets smaller and the joint stiffness K gets larger.
At this point it should be mentioned that the joint stiffnesses of a *typical*[6] flexible joint robot can indeed be assumed to be quite large[7] (but not negligible[8]). By a negative feedback of the joint torque τ the apparent inertia (of the rotor) can now be scaled down, which means that the closed loop system reacts to external forces and torques as if the rotor inertia were smaller. The desired dynamical behaviour will be approximated the better, the smaller the apparent rotor inertia is. This approach, as suggested in figure 2 intuitively, will be put in concrete terms in the following section for the model of a flexible joint robot.

[4] The motors are herein modelled as ideal torque sources without electrical dynamics.
[5] In Figure 1 represented in a simplified form with a constant mass M.
[6] like e.g. the DLR-Light-Weight-Robots
[7] E.g.: For the lower joints of the DLR-Light-Weight-Robots these values ly in the range $10.000 - 15.000 Nm/rad$.
[8] In the second paper [3] a method for compensating the influence of the spring K will also be presented.

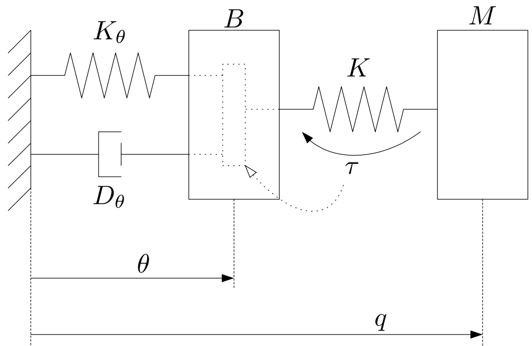

Fig. 2. Motor Position Based PD-Control of a Single Joint.

III. Controller Design

A. Considered Model

In the following a flexible joint robot model is assumed as was proposed by Spong in [9]:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{ext} \ , \qquad (1)$$

$$\boldsymbol{B}\ddot{\boldsymbol{\theta}} + \boldsymbol{\tau} = \boldsymbol{\tau}_m \ . \qquad (2)$$

Herein $\boldsymbol{q} \in \Re^n$ represents the vector of link side joint angles and $\boldsymbol{\theta} \in \Re^n$ the vector of motor angles. The joint torques $\boldsymbol{\tau} \in \Re^n$ are determined by the linear relationship $\boldsymbol{\tau} = \boldsymbol{K}(\boldsymbol{\theta} - \boldsymbol{q})$, in which $\boldsymbol{K} \in \Re^{n\times n}$ is a diagonal matrix containing the individual joint stiffnesses K_i as diagonal elements $\boldsymbol{K} = diag(K_i)$. $\boldsymbol{B} \in \Re^{n\times n}$ is a diagonal matrix, which consists of the rotor inertias B_i. Furthermore $\boldsymbol{M}(\boldsymbol{q}) \in \Re^{n\times n}$ is the manipulators (link side) mass matrix and $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ represents the centrifugal and Coriolis-terms of the link side rigid body part of the model. The vector of gravity torques is given by $\boldsymbol{g}(\boldsymbol{q}) \in \Re^n$. The motor torques $\boldsymbol{\tau}_m \in \Re^n$ are considered as the input signals for the control. Finally, the external forces and torques which act on the robot are summarized in the torque vector $\boldsymbol{\tau}_{ext} \in \Re^n$.

In case that the robot has mixed, rotational and prismatic, joints it is assumed that the elements of the vector $\boldsymbol{q}$ are normalized such that they all have the same physical unit. This can be achieved by multiplying the joint variables of the prismatic joints by a factor $1/r_i$, where r_i has the physical unit of a length. The normalization of $\boldsymbol{q}$ allows the use of the Euclidean norm $||\cdot||_2$ for vectors and the induced 2-norm $||\cdot||_{i2}$ for matrices without leading to physical inconsistencies[9].

At this point also some well known properties of the robot model shall be mentioned, which will be utilized in the following sections:

Property 1: The mass matrix is symmetric and p.d.:

$$\boldsymbol{M}(\boldsymbol{q}) = \boldsymbol{M}(\boldsymbol{q})^T > 0 \quad \forall \boldsymbol{q} \in \Re^n \ .$$

[9]The normalization is equivalent to using the norm $||\boldsymbol{q}|| = \sqrt{\sum_{i=1}^n \gamma_i^2 q_i^2}$ (with $\gamma_i = 1/r_i$ for prismatic joints and $\gamma_i = 1$ for rotational joints) instead of the Euclidean norm.

Property 2: $\dot{\boldsymbol{M}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is skew symmetric:

$$\boldsymbol{y}^T(\dot{\boldsymbol{M}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}))\boldsymbol{y} = 0 \quad \forall \boldsymbol{y},\boldsymbol{q},\dot{\boldsymbol{q}} \in \Re^n \ .$$

Property 3: The gravity torques $\boldsymbol{g}(\boldsymbol{q})$ are given by the differential of a potential function $V_g(\boldsymbol{q})$, hence $\boldsymbol{g}(\boldsymbol{q}) = (\partial V_g(\boldsymbol{q})/\partial \boldsymbol{q})^T$, and there exists an $\alpha > 0$ such[10] that

$$||\partial \boldsymbol{g}(\boldsymbol{q})/\partial \boldsymbol{q}||_{i2} < \alpha \qquad \forall \boldsymbol{q} \in \Re^n$$

holds. The inequality implies that for all $\boldsymbol{q}_1, \boldsymbol{q}_2 \in \Re^n$ the following holds:

$$|V_g(\boldsymbol{q}_2) - V_g(\boldsymbol{q}_1) - (\boldsymbol{q}_2 - \boldsymbol{q}_1)^T \boldsymbol{g}(\boldsymbol{q}_1)| \le \frac{1}{2}\alpha ||\boldsymbol{q}_2 - \boldsymbol{q}_1||_2^2 \ .$$

Remark 1: A further assumption on the dynamical model (1)-(2) is that the joint stiffnesses satisfy[11] $K_i > \alpha$. Notice that this assumption is not restrictive at all! Intuitively speaking it states nothing else than that the manipulator should be designed properly, which means that the joint springs are sufficiently stiff such that they can prevent the manipulator from falling down under the action of its weight.

B. Controller Design for a Joint Space Impedance

The scaling of the apparent rotor inertia from $\boldsymbol{B}$ to $\boldsymbol{B}_\theta$ can, as was already described in the last section, be achieved by a joint torque feedback

$$\boldsymbol{\tau}_m = \boldsymbol{B}\boldsymbol{B}_\theta^{-1}\boldsymbol{u} + (\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}_\theta^{-1})\boldsymbol{\tau} \ , \qquad (3)$$

wherein $\boldsymbol{u}$ is a new input variable. In case that the desired impedance behaviour is defined (w.r.t. joint coordinates) in form of a positive definite stiffness matrix $\boldsymbol{K}_\theta$ and damping matrix $\boldsymbol{D}_\theta$ as well as a desired configuration $\boldsymbol{\theta}_s$, a motor position based PD-controller

$$\boldsymbol{u} = -\boldsymbol{K}_\theta(\boldsymbol{\theta} - \boldsymbol{\theta}_s) - \boldsymbol{D}_\theta\dot{\boldsymbol{\theta}} \qquad (4)$$

can then be used. Therefore one gets the following closed loop equations:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{ext} \ , \qquad (5)$$

$$\boldsymbol{B}_\theta\ddot{\boldsymbol{\theta}} + \boldsymbol{D}_\theta\dot{\boldsymbol{\theta}} + \boldsymbol{K}_\theta(\boldsymbol{\theta} - \boldsymbol{\theta}_s) + \boldsymbol{\tau} = \boldsymbol{0} \ . \qquad (6)$$

C. Generalization to Cartesian coordinates

Usually the desired impedance behaviour is defined with respect to Cartesian coordinates $\boldsymbol{x} \in \Re^m$, which describe the position and orientation of the robots endeffector, rather than in joint space. In the following it is assumed that the forward kinematics mapping from the joint space coordinates $\boldsymbol{q}$ to the Cartesian coordinates $\boldsymbol{x}_q = \boldsymbol{f}(\boldsymbol{q})$ as well as the Jacobian matrix $\boldsymbol{J}(\boldsymbol{q}) = \frac{\partial \boldsymbol{f}(\boldsymbol{q})}{\partial \boldsymbol{q}}$ are known.

[10]The physical unit of α is a joint stiffness.

[11]Notice that in general, the fulfilment of the condition $K_i > \alpha$ depends on the choice of the above mentioned normalization of the vector $\boldsymbol{q}$ via the parameters r_i (or, equivalently, on the norm used to evaluate the condition). From the equivalence of norms on $\Re^n$ (see e.g. [14]) it follows that the convergence properties which will be derived in the paper hold as long as a particular scaling can be found, so that $K_i > \alpha$.

The controller from (4) can then be generalized to Cartesian coordinates easily, if one uses the motor angles $\boldsymbol{\theta}$ instead of the link side angles $\boldsymbol{q}$ in the forward kinematics. The desired stiffness and damping matrices are given in form of positive definite matrices $\boldsymbol{K}_x$ and $\boldsymbol{D}_x$. Then the feedback law

$$\begin{aligned} \boldsymbol{u} &= -\boldsymbol{J}(\boldsymbol{\theta})^T(\boldsymbol{K}_x\tilde{\boldsymbol{x}}(\boldsymbol{\theta}) + \boldsymbol{D}_x\dot{\boldsymbol{x}}) \ , && (7)\\ \tilde{\boldsymbol{x}}(\boldsymbol{\theta}) &= \boldsymbol{f}(\boldsymbol{\theta}) - \boldsymbol{x}_s && (8)\\ \dot{\boldsymbol{x}} &= \boldsymbol{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} && (9)\end{aligned}$$

generalizes (4) to Cartesian coordinates. Herein, $\boldsymbol{x}_s$ is the *virtual*[12] motor side position in Cartesian coordinates. Notice that in the design of $\boldsymbol{x}_s$ the static (i.e. in equilibrium state for $\boldsymbol{\tau}_{ext} = \boldsymbol{0}$) difference of the motor and link side angles due to gravity should be considered. This means that, for a given link side position $\boldsymbol{q}_s$ which corresponds to the desired Cartesian position $\boldsymbol{x}_{q,s} = \boldsymbol{f}(\boldsymbol{q}_s)$, $\boldsymbol{x}_s$ should be chosen as: $\boldsymbol{x}_s = \boldsymbol{f}(\boldsymbol{q}_s + \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q}_s))$.
The controller in (7) leads then, together with (3), to the closed loop system:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{ext} \ , \qquad (10)$$

$$\boldsymbol{B}_\theta\ddot{\boldsymbol{\theta}} + \boldsymbol{J}(\boldsymbol{\theta})^T(\boldsymbol{K}_x\tilde{\boldsymbol{x}}(\boldsymbol{\theta}) + \boldsymbol{D}_x\dot{\boldsymbol{x}}) + \boldsymbol{\tau} = \boldsymbol{0} \ . \qquad (11)$$

IV. Gravity Compensation

In the derivations so far the gravity torque $\boldsymbol{g}(\boldsymbol{q})$ was not considered in the controller. It has been shown in [12] that for a motor position based PD-controller a feedforward term of the gravity torques in the desired steady state can be used. This indeed leads in case of a position controller usually to a good performance, because the deviations from the steady state position can be kept small. In case of an impedance controller however this is not true. Here a pure feedforward action for the gravity compensation does not give satisfactory results, because large deviations from the steady state positions may occur in case of a small desired stiffness $\boldsymbol{K}_x$. Therefore in the following a compensation term is constructed which is based solely on the motor position and can compensate for the link side gravity torques (in a *quasi-stationary* fashion). Consider first the set $\Omega := \{(\boldsymbol{q},\boldsymbol{\theta}) \mid \boldsymbol{K}(\boldsymbol{\theta}-\boldsymbol{q}) = \boldsymbol{g}(\boldsymbol{q})\}$ of stationary points (for $\boldsymbol{\tau}_{ext} = \boldsymbol{0}$) for which the torque due to the joint elasticity counterbalances the link side gravity torque. The goal of the gravity compensation is now to construct a compensation term $\bar{\boldsymbol{g}}(\boldsymbol{\theta})$ such that in Ω

$$\bar{\boldsymbol{g}}(\boldsymbol{\theta}) = \boldsymbol{g}(\boldsymbol{q}) \quad \forall(\boldsymbol{q},\boldsymbol{\theta}) \in \Omega \qquad (12)$$

holds, which means that the gravity compensation term counterbalances the link side gravity torque in all stationary points. Notice that for any point $(\boldsymbol{q}_0,\boldsymbol{\theta}_0) \in \Omega$ the motor position can now be expressed uniquely as a function of the link side position:

$$\boldsymbol{\theta}_0 = \boldsymbol{q}_0 + \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q}_0) := \boldsymbol{h}(\boldsymbol{q}_0) \ . \qquad (13)$$

[12]In impedance control the desired steady state position for the case of free motion is usually called *virtual* position.

Furthermore, by the use of the contraction theorem (see Remark 2 below for more details on this) it can be shown that the inverse function to $\boldsymbol{h}(\boldsymbol{q}_0)$ exists. Then

$$\boldsymbol{q}_0 = \boldsymbol{h}^{-1}(\boldsymbol{\theta}_0) := \bar{\boldsymbol{q}}(\boldsymbol{\theta}_0) \qquad (14)$$

can be used for the construction of a gravity compensation term of the form $\bar{\boldsymbol{g}}(\boldsymbol{\theta}) := \boldsymbol{g}(\bar{\boldsymbol{q}}(\boldsymbol{\theta}))$ which clearly fullfills (12).

Remark 2: While in general the inverse function $\boldsymbol{h}^{-1}(\boldsymbol{\theta}_0)$ can not be computed directly in practice, it is possible to approximate it with arbitrary accuracy by iteration. Notice therefore that (since $K_i > \alpha$ from Remark 1) the mapping $\boldsymbol{T}(\boldsymbol{q}) := \boldsymbol{\theta}_0 - \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q})$ is a contraction and thus has a unique fixpoint $\boldsymbol{q}^* = \boldsymbol{T}(\boldsymbol{q}^*) = \boldsymbol{q}_0$. The iteration

$$\hat{\boldsymbol{q}}_{n+1} = \boldsymbol{T}(\hat{\boldsymbol{q}}_n) \qquad (15)$$

converges then for every starting point (e.g. $\hat{\boldsymbol{q}}_0 = \boldsymbol{\theta}_0$) to this fixpoint, as follows from the contraction mapping theorem (see e.g. [14]):

$$\lim_{n\to\infty} \hat{\boldsymbol{q}}_n = \boldsymbol{q}^* = \boldsymbol{h}^{-1}(\boldsymbol{\theta}_0) \ . \qquad (16)$$

In the following it is therefore assumed that the inverse function $\boldsymbol{h}^{-1}(\boldsymbol{\theta}_0)$ is known exactly, although it can only be approximated in practice. It should also be noted that usually already one or two iteration steps lead to quite satisfactory results in this approximation. Notice also that by a first order approximation with $\hat{\boldsymbol{q}}_0 = \boldsymbol{q}_s$ one would obtain the online gravity compensation term of [15].
From the construction of the gravity compensation term it also follows that $\bar{\boldsymbol{g}}(\boldsymbol{\theta})$ is given as the differential of a potential function $V_{\bar{g}}(\boldsymbol{\theta})$. A detailed derivation of this potential function is given in the appendix. However, in order to show that the function is given by

$$\begin{aligned} V_{\bar{g}}(\boldsymbol{\theta}) &= V_g(\bar{\boldsymbol{q}}(\boldsymbol{\theta})) + V_k(\boldsymbol{\theta} - \bar{\boldsymbol{q}}(\boldsymbol{\theta})) && (17)\\ &= V_g(\bar{\boldsymbol{q}}(\boldsymbol{\theta})) + \frac{1}{2}\bar{\boldsymbol{g}}(\boldsymbol{\theta})^T\boldsymbol{K}^{-1}\bar{\boldsymbol{g}}(\boldsymbol{\theta}) && (18)\end{aligned}$$

where $V_k(\boldsymbol{\theta}-\boldsymbol{q}) = 1/2(\boldsymbol{\theta}-\boldsymbol{q})^T\boldsymbol{K}(\boldsymbol{\theta}-\boldsymbol{q})$ is the potential energy of the joint elasticity we compute its differential (with $\boldsymbol{r}(\boldsymbol{\theta}) := \boldsymbol{\theta} - \bar{\boldsymbol{q}}(\boldsymbol{\theta})$) as:

$$\begin{aligned} \frac{\partial V_{\bar{\boldsymbol{g}}}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} &= \frac{\partial V_g(\bar{\boldsymbol{q}})}{\partial\bar{\boldsymbol{q}}}\frac{\partial\bar{\boldsymbol{q}}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} + \frac{\partial V_k(\boldsymbol{r})}{\partial\boldsymbol{r}}\frac{\partial\boldsymbol{r}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} && (19)\\ &= \frac{\partial V_g(\bar{\boldsymbol{q}})}{\partial\bar{\boldsymbol{q}}} = \boldsymbol{g}(\bar{\boldsymbol{q}}(\boldsymbol{\theta}))^T = \bar{\boldsymbol{g}}(\boldsymbol{\theta})^T && (20)\end{aligned}$$

Herein, the property $\partial V_k(\boldsymbol{r})/\partial\boldsymbol{r} = \partial V_g(\bar{\boldsymbol{q}})/\partial\bar{\boldsymbol{q}}$ was used, which follows directly from the definition of the function $\bar{\boldsymbol{q}}(\boldsymbol{\theta})$ in (14).
The complete control law with gravity compensation is given now by (3) together with

$$\boldsymbol{u} = -\boldsymbol{J}(\boldsymbol{\theta})^T(\boldsymbol{K}_x\tilde{\boldsymbol{x}}(\boldsymbol{\theta}) + \boldsymbol{D}_x\dot{\boldsymbol{x}}) + \bar{\boldsymbol{g}}(\boldsymbol{\theta}) \ , \qquad (21)$$

and leads to the closed loop system equations:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{ext} \ , \qquad (22)$$

$$\boldsymbol{B}_\theta\ddot{\boldsymbol{\theta}} + \boldsymbol{J}(\boldsymbol{\theta})^T(\boldsymbol{K}_x\tilde{\boldsymbol{x}}(\boldsymbol{\theta}) + \boldsymbol{D}_x\dot{\boldsymbol{x}}) + \boldsymbol{\tau} = \bar{\boldsymbol{g}}(\boldsymbol{\theta}) \ . \qquad (23)$$

V. Analysis

In this section it will first be shown that in case of a globally bounded potential function $V_g(\boldsymbol{q})$ the closed loop system can be written as the interconnection of two passive subsystems. Additionally, a proof of asymptotical stability is given for the general case.
For the passivity analysis it will be assumed that there exists a real $\beta > 0$, such that

$$|V_g(\boldsymbol{q})| \quad < \quad \beta \qquad \forall \boldsymbol{q} \in \Re^n \tag{24}$$

holds. This is for instance satisfied for all robots with solely rotational joints (i.e. without prismatic joints). Due to property 3 also the gravity torque vector $\boldsymbol{g}(\boldsymbol{q})$ will then be globally bounded. Furthermore from (24) also the boundedness of $V_{\bar{g}}(\boldsymbol{\theta})$ and $\bar{\boldsymbol{g}}(\boldsymbol{\theta})$ follows. Notice that requirement of a bounded gravity potential is only needed for the passivity analysis, while the proof on asymptotic stability is valid for a general potential.

A. Passivity Analysis

A system $\dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{z}, \boldsymbol{u}), \boldsymbol{y} = \boldsymbol{y}(\boldsymbol{z}, \boldsymbol{u})$ with state $\boldsymbol{z} \in \Re^n$, input $\boldsymbol{u} \in \Re^m$ and output $\boldsymbol{y} \in \Re^m$ is said to be *passive*, if for an admissible input $\boldsymbol{u}(t)$ the *energy* that can be extracted from the system in an arbitrary time interval $[t_0, t_1]$ is bounded from below [13]:

$$\exists c \in \Re : \qquad \int_{t_0}^{t_1} \boldsymbol{u}(t)^T \boldsymbol{y}(t) dt \geq c \tag{25}$$

A sufficient condition therefore is given by the existence of a continuous (in $\boldsymbol{z}$) function $S_z(\boldsymbol{z})$ (storage function [8], [13]), which is bounded from below and for which the derivative with respect to time along the solutions of the system satisfies the inequality:

$$\dot{S}_z(\boldsymbol{z}) = \frac{\partial S_z(\boldsymbol{z})}{\partial \boldsymbol{z}} \boldsymbol{f}(\boldsymbol{z}, \boldsymbol{u}) \leq \boldsymbol{u}^T \boldsymbol{y} \ . \tag{26}$$

In the following it will now be shown that the system (22)-(23), as outlined in figure 3, consists of two passive subsystems. It is often assumed that also the environment of the robot can be described by a passive mapping ($\dot{\boldsymbol{q}} \rightarrow -\boldsymbol{\tau}_{ext}$).
The passivity of (22), as a mapping $(\boldsymbol{\tau} + \boldsymbol{\tau}_{ext}) \rightarrow \dot{\boldsymbol{q}}$, is well known due to purely physical reasons and can be shown with the storage function

$$S_q = \frac{1}{2} \dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q}) \dot{\boldsymbol{q}} + V_g(\boldsymbol{q}) \tag{27}$$

for which the derivative along the solutions of (22) is given by[13]:

$$\dot{S}_q = \dot{\boldsymbol{q}}^T (\boldsymbol{\tau} + \boldsymbol{\tau}_{ext}) \ . \tag{28}$$

In a similar way the passivity of (23), as a mapping $\dot{\boldsymbol{q}} \rightarrow -\boldsymbol{\tau}$, can be shown with the storage function:

$$\begin{aligned} S_\theta \quad = \quad & \frac{1}{2} \dot{\boldsymbol{\theta}}^T \boldsymbol{B}_\theta \dot{\boldsymbol{\theta}} + \frac{1}{2} (\boldsymbol{\theta} - \boldsymbol{q})^T \boldsymbol{K} (\boldsymbol{\theta} - \boldsymbol{q}) \\ & + \frac{1}{2} \tilde{\boldsymbol{x}}(\boldsymbol{\theta})^T \boldsymbol{K}_x \tilde{\boldsymbol{x}}(\boldsymbol{\theta}) - V_{\bar{g}}(\boldsymbol{\theta}) \end{aligned} \tag{29}$$

[13]This is due to Property 2.

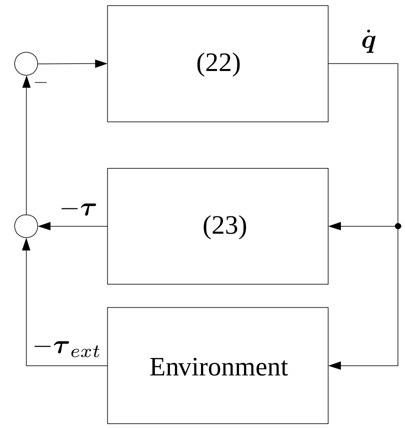

Fig. 3. System Representation as Interconnection of Passive Subsystems.

The derivative of S_θ along the solutions of (23) is then given by:

$$\dot{S}_\theta = -\dot{\boldsymbol{x}}^T \boldsymbol{D}_x \dot{\boldsymbol{x}} - \dot{\boldsymbol{q}}^T \boldsymbol{\tau} \ . \tag{30}$$

The desired passivity properties follow then directly from (28) and (30).

Remark 3: It should also be mentioned that the passivity properties would still be valid if the PD-controller from (7) is replaced by any other passive (w.r.t. $\dot{\boldsymbol{\theta}} \rightarrow \boldsymbol{u}$) controller.

B. Stability Proof for $\boldsymbol{\tau}_{ext} = \boldsymbol{0}$

The following stability analysis is restricted to the nonredundant case ($m = n$). Additional to that, singular configurations have to be avoided. Thus the further analysis is restricted to an area in the workspace in which the Jacobian $\boldsymbol{J}(\boldsymbol{\theta})$ is nonsingular and in which the inverse mapping to $\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{\theta})$ can be solved uniquely.
In the following it will be shown that the closed loop system is asymptotically stable for the case of free motion (i.e. $\boldsymbol{\tau}_{ext} = \boldsymbol{0}$).

1) Determination of the steady state: The steady state condition of the system (22)-(23) is given by

$$\boldsymbol{K}(\boldsymbol{\theta}_s - \boldsymbol{q}_s) = \boldsymbol{g}(\boldsymbol{q}_s) \ , \tag{31}$$

$$\boldsymbol{K}(\boldsymbol{\theta}_s - \boldsymbol{q}_s) + \boldsymbol{J}(\boldsymbol{\theta}_s)^T \boldsymbol{K}_x \tilde{\boldsymbol{x}}(\boldsymbol{\theta}_s) = \bar{\boldsymbol{g}}(\boldsymbol{\theta}_s) \ . \tag{32}$$

Herein the matrix $\boldsymbol{K}_x$ is positive definite. Due to (12) it follows that

$$\boldsymbol{J}(\boldsymbol{\theta}_s)^T \boldsymbol{K}_x \tilde{\boldsymbol{x}}(\boldsymbol{\theta}_s) \quad = \quad \boldsymbol{0} \tag{33}$$

must be satisfied in steady state. The stability analysis is, as already mentioned above, restricted to an area in which this condition can be solved uniquely for $\boldsymbol{\theta}_s$. The steady state is then given by:

$$\begin{aligned} \tilde{\boldsymbol{x}}(\boldsymbol{\theta}_s) \quad &= \quad \boldsymbol{0} \Rightarrow \boldsymbol{\theta}_s = \boldsymbol{f}^{-1}(\boldsymbol{x}_s) \ , \\ \boldsymbol{q}_s \quad &= \quad \boldsymbol{h}^{-1}(\boldsymbol{\theta}_s) \ , \\ \dot{\boldsymbol{q}}_s \quad &= \quad \dot{\boldsymbol{\theta}}_s = \boldsymbol{0} \ . \end{aligned}$$

2) Lyapunov-Function: Consider the following function $V(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ as a candidate Lyapunov function:

$$V(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = S_q + S_\theta \ . \tag{34}$$

In steady state the following holds (due to (18))[14]:

$$\begin{aligned} V(\boldsymbol{q}_s, \boldsymbol{0}, \boldsymbol{\theta}_s, \boldsymbol{0}) &= V_g(\boldsymbol{q}_s) - V_{\bar{g}}(\boldsymbol{\theta}_s) \\ &\quad + \frac{1}{2}(\boldsymbol{\theta}_s - \boldsymbol{q}_s)^T \boldsymbol{K}(\boldsymbol{\theta}_s - \boldsymbol{q}_s) \\ &= 0 \ . \end{aligned}$$

The kinetic part of $V(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$

$$V_{kin} = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{\theta}}^T \boldsymbol{B}_\theta \dot{\boldsymbol{\theta}}$$

is positive definite with respect to $\dot{\boldsymbol{q}}$ und $\dot{\boldsymbol{\theta}}$, because the mass matrix is positive definite. In order to show that $V(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ is positive definite, it is then sufficient to show that the potential part

$$V_{pot}(\boldsymbol{q}, \boldsymbol{\theta}) = V(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) - V_{kin} \tag{35}$$

is positive definite with respect to $\boldsymbol{q}$ and $\boldsymbol{\theta}$.

Consider at first only the part of the potential energy due to $\boldsymbol{K}$. In the following $\bar{\boldsymbol{q}}$ is written instead of $\bar{\boldsymbol{q}}(\boldsymbol{\theta})$ in order to simplify the notation.

$$\begin{aligned} V_K &:= \frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{q})^T \boldsymbol{K}(\boldsymbol{\theta} - \boldsymbol{q}) \\ &= \frac{1}{2}(\boldsymbol{\theta} - \bar{\boldsymbol{q}} + \bar{\boldsymbol{q}} - \boldsymbol{q})^T \boldsymbol{K}(\boldsymbol{\theta} - \bar{\boldsymbol{q}} + \bar{\boldsymbol{q}} - \boldsymbol{q}) \\ &= \frac{1}{2}\boldsymbol{g}(\bar{\boldsymbol{q}})^T \boldsymbol{K}^{-1}\boldsymbol{g}(\bar{\boldsymbol{q}}) + \frac{1}{2}(\bar{\boldsymbol{q}} - \boldsymbol{q})^T \boldsymbol{K}(\bar{\boldsymbol{q}} - \boldsymbol{q}) \\ &\quad + (\bar{\boldsymbol{q}} - \boldsymbol{q})^T \boldsymbol{g}(\bar{\boldsymbol{q}}) \end{aligned}$$

Herein the relationship $\boldsymbol{K}(\boldsymbol{\theta} - \bar{\boldsymbol{q}}) = \boldsymbol{g}(\bar{\boldsymbol{q}})$ was used which holds, contrary to (12), always (i.e. not only in Ω). The potential energy can then be written (with (18)) as follows:

$$\begin{aligned} V_{pot}(\boldsymbol{q}, \boldsymbol{\theta}) &= V_K + \frac{1}{2}\tilde{\boldsymbol{x}}(\boldsymbol{\theta})^T \boldsymbol{K}_x \tilde{\boldsymbol{x}}(\boldsymbol{\theta}) + \\ &\quad V_g(\boldsymbol{q}) - V_{\bar{g}}(\boldsymbol{\theta}) \\ &= V_K + \frac{1}{2}\tilde{\boldsymbol{x}}(\boldsymbol{\theta})^T \boldsymbol{K}_x \tilde{\boldsymbol{x}}(\boldsymbol{\theta}) + \\ &\quad V_g(\boldsymbol{q}) - V_g(\bar{\boldsymbol{q}}) - \frac{1}{2}\boldsymbol{g}(\bar{\boldsymbol{q}})^T \boldsymbol{K}^{-1}\boldsymbol{g}(\bar{\boldsymbol{q}}) \end{aligned}$$

Due to property 3 the following inequalities hold:

$$\begin{aligned} V_{pot} &\geq \frac{1}{2}(\bar{\boldsymbol{q}} - \boldsymbol{q})^T \boldsymbol{K}(\bar{\boldsymbol{q}} - \boldsymbol{q}) + \frac{1}{2}\tilde{\boldsymbol{x}}(\boldsymbol{\theta})^T \boldsymbol{K}_x \tilde{\boldsymbol{x}}(\boldsymbol{\theta}) \\ &\quad - |V_g(\boldsymbol{q}) - V_g(\bar{\boldsymbol{q}}) - (\boldsymbol{q} - \bar{\boldsymbol{q}})^T \boldsymbol{g}(\bar{\boldsymbol{q}})| \\ &\geq \frac{1}{2}(\bar{\boldsymbol{q}} - \boldsymbol{q})^T (\boldsymbol{K} - \alpha \boldsymbol{I})(\bar{\boldsymbol{q}} - \boldsymbol{q}) + \frac{1}{2}\tilde{\boldsymbol{x}}(\boldsymbol{\theta})^T \boldsymbol{K}_x \tilde{\boldsymbol{x}}(\boldsymbol{\theta}) \end{aligned}$$

The right hand side of the last inequality is nonnegative for all $(\boldsymbol{q}, \boldsymbol{\theta})$, since $K_i > \alpha$ (from Remark 1). The area in which the term $\tilde{\boldsymbol{x}}(\boldsymbol{\theta})^T \boldsymbol{K}_x \tilde{\boldsymbol{x}}(\boldsymbol{\theta})$ is positive definite (in $\boldsymbol{\theta}$) finally determines the area in which the Lyapunov function is positive definite. For the case of a general forward kinematics only local statements can be made therefore.

[14]Remember that in steady state $\boldsymbol{q}_s = \bar{\boldsymbol{q}}(\boldsymbol{\theta}_s)$.

3) Derivative of the Lyapunov-Function: The derivative of $V(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ along the solutions of the system (22)-(23) (for $\boldsymbol{\tau}_{ext} = \boldsymbol{0}$) is given by:

$$\dot{V}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \dot{S}_q + \dot{S}_\theta = -\dot{\boldsymbol{x}}^T \boldsymbol{D}_x \dot{\boldsymbol{x}} \ . \tag{36}$$

Due to the fact that the matrix $\boldsymbol{D}_x$ is positive definite, it can then be concluded that the equilibrium point is stable. Furthermore asymptotic stability can be shown by the use of the invariance principle of LaSalle. Therefore the system state will converge into the largest positively invariant set for which $\dot{\boldsymbol{x}} = \boldsymbol{0}$ holds. From the system equations it follows that there does not exist any trajectory for which $\dot{\boldsymbol{x}} = \boldsymbol{0}$ holds exept for the restriction to the equilibrium point[15]. Therefore asymptotic stability can be concluded.

4) Some Additional Remarks:

- In contrast to any previous works, no restrictions are imposed on the p.d. matrix $\boldsymbol{K}_x$ for stability, meaning that the stiffness can be commanded arbitrarily close to zero.
- *The redundant case:*
  Notice that by the same argumentation as above one can in principle also show convergence of the Cartesian error $\tilde{\boldsymbol{x}} \to \boldsymbol{0}$ in the redundant case $m < n$ (as long as singular configurations are avoided). However, it is then of course necessary to add also a nullspace damping term in order to ensure $\dot{\boldsymbol{\theta}} \to \boldsymbol{0}$ (see e.g. [7]).
- *Global Analysis in Joint Space*
  It should also be mentioned that the stability analysis would have led to a globally valid statement in case of a joint space impedance controller (as in Section III-B). The reason for the fact that the analysis is only valid locally for the Cartesian impedance controller is explained hereafter. In the previous analysis the motor angles θ_i of the rotational joints are seen as elements of $\Re$ instead of the more appropriate manifold S^1 (the unit circle). Any set of Cartesian coordinates which describe the position and orientation of the endeffector therefore must necessarily be periodic in $\boldsymbol{\theta}$. Furthermore it is well known that, due to topological reasons[16], it is not possible to design a potential function in Cartesian space (SE(3)) which has a single global extremum. Besides that, the manipulator singularities (as well as additional representation singularities of the coordinate function $\boldsymbol{f}$) represent further restrictions to the analysis of the Cartesian controller.

VI. Conclusion

In this paper it has been shown that the feedback of joint torques for a flexible joint robot can be interpreted physically as a scaling of the motor inertia. This new interpretation allows in principle the combination of a torque feedback action with any controller designed for flexible joint robots. In this paper

[15]Notice again that the analysis is restricted to a workspace in which the Jacobian is nonsingular.

[16]and Morse's theory

the case of a Cartesian impedance controller was treated in detail. A stability analysis of the presented controller was given which was based on the passivity properties of the torque controlled motor dynamics and the impedance controller. The design of an appropriate gravity compensation term poses an additional problem in case of a robot with nonnegligible joint flexibility. Therefore a compensation term was proposed which is based only on the measurement of the motor side position. In order to achieve a high control performance in practice, many extentions to the presented controller are possible. They are the topics of the second paper [3] in which it also will be shown how the controller must be extended in case of nonnegligible joint damping (i.e. damping in parallel to the joint stiffness K). Furthermore also some experimental results with the presented controller will be given in the second part.

APPENDIX

In this appendix the potential function $V_{\bar{g}}(\boldsymbol{\theta})$ for the gravity compensation term $\bar{\boldsymbol{g}}(\boldsymbol{\theta})$ will be derived. Remember that for the construction of $\bar{\boldsymbol{g}}(\boldsymbol{\theta}) := \boldsymbol{g}(\bar{\boldsymbol{q}}(\boldsymbol{\theta}))$ in Section IV the function $\bar{\boldsymbol{q}}(\boldsymbol{\theta}) := \boldsymbol{h}^{-1}(\boldsymbol{\theta})$, i.e. the inverse of the function $\boldsymbol{h}(\boldsymbol{q}) := \boldsymbol{q} + \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q})$, was used. Existence and uniqueness of $\boldsymbol{h}^{-1}(\boldsymbol{\theta})$ were already established in Remark 2.

In the following the Jacobian matrix $\partial\bar{\boldsymbol{q}}(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$ will be needed. Consider first the Jacobian matrix of the function $\boldsymbol{h}(\boldsymbol{q})$:

$$\frac{\partial \boldsymbol{h}(\boldsymbol{q})}{\partial \boldsymbol{q}} = \left(\boldsymbol{I} + \boldsymbol{K}^{-1} \frac{\partial \boldsymbol{g}(\boldsymbol{q})}{\partial \boldsymbol{q}} \right) . \tag{37}$$

Due to $\boldsymbol{h}(\bar{\boldsymbol{q}}(\boldsymbol{\theta})) = \boldsymbol{\theta}$, the Jacobian matrix $\frac{\partial \bar{\boldsymbol{q}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$ must be

$$\frac{\partial \bar{\boldsymbol{q}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \left(\boldsymbol{I} + \boldsymbol{K}^{-1} \frac{\partial \boldsymbol{g}(\bar{\boldsymbol{q}})}{\partial \bar{\boldsymbol{q}}} \right)^{-1}_{\bar{\boldsymbol{q}} = \bar{\boldsymbol{q}}(\boldsymbol{\theta})} . \tag{38}$$

The potential function $V_{\bar{g}}(\boldsymbol{\theta})$ clearly can be written in the form

$$V_{\bar{g}}(\boldsymbol{\theta}) = V_{\bar{g}}(\boldsymbol{h}(\bar{\boldsymbol{q}}(\boldsymbol{\theta}))) =: V_{\bar{g}h}(\bar{\boldsymbol{q}}(\boldsymbol{\theta})) \tag{39}$$

For the differential $\partial V_{\bar{g}}(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$ one obtains

$$\frac{\partial V_{\bar{g}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \left(\frac{\partial V_{\bar{g}h}(\bar{\boldsymbol{q}})}{\partial \bar{\boldsymbol{q}}} \right)_{\bar{\boldsymbol{q}} = \bar{\boldsymbol{q}}(\boldsymbol{\theta})} \frac{\partial \bar{\boldsymbol{q}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \tag{40}$$

By substituting (as desired) $\frac{\partial V_{\bar{g}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \boldsymbol{g}(\bar{\boldsymbol{q}}(\boldsymbol{\theta}))^T$ and $\frac{\partial \bar{\boldsymbol{q}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$ from (38) one gets

$$\begin{aligned} \frac{\partial V_{\bar{g}h}(\bar{\boldsymbol{q}})}{\partial \bar{\boldsymbol{q}}} &= \boldsymbol{g}(\bar{\boldsymbol{q}})^T \left(\boldsymbol{I} + \boldsymbol{K}^{-1} \frac{\partial \boldsymbol{g}(\bar{\boldsymbol{q}})}{\partial \bar{\boldsymbol{q}}} \right) , \\ &= \boldsymbol{g}(\bar{\boldsymbol{q}})^T + \boldsymbol{g}(\bar{\boldsymbol{q}})^T \boldsymbol{K}^{-1} \frac{\partial \boldsymbol{g}(\bar{\boldsymbol{q}})}{\partial \bar{\boldsymbol{q}}} . \end{aligned}$$

This differential can then be integrated to $V_{\bar{g}h}(\bar{\boldsymbol{q}}) = V_g(\bar{\boldsymbol{q}}) + \frac{1}{2}\boldsymbol{g}(\bar{\boldsymbol{q}})^T\boldsymbol{K}^{-1}\boldsymbol{g}(\bar{\boldsymbol{q}}) + \boldsymbol{c}$, with an arbitrary constant $\boldsymbol{c} \in \Re^n$. Setting $\boldsymbol{c} = \boldsymbol{0}$ leads to the gravity compensation potential $V_{\bar{g}}(\boldsymbol{\theta})$ from Section IV:

$$V_{\bar{g}}(\boldsymbol{\theta}) = V_g(\bar{\boldsymbol{q}}(\boldsymbol{\theta})) + \frac{1}{2}\boldsymbol{g}(\bar{\boldsymbol{q}}(\boldsymbol{\theta}))^T \boldsymbol{K}^{-1} \boldsymbol{g}(\bar{\boldsymbol{q}}(\boldsymbol{\theta})) . \tag{41}$$

Notice also that the complete potential function $V_c(\boldsymbol{q}, \boldsymbol{\theta}) = V_g(\boldsymbol{q}) + V_k(\boldsymbol{\theta} - \boldsymbol{q}) - V_{\bar{g}}(\boldsymbol{\theta})$ vanishes at all stationary points

$$V_c(\boldsymbol{q}, \boldsymbol{\theta}) = 0 \quad \forall \ (\boldsymbol{q}, \boldsymbol{\theta}) \in \Omega . \tag{42}$$

Therefore $V_{\bar{g}}(\boldsymbol{\theta})$ can also be written as

$$V_{\bar{g}}(\boldsymbol{\theta}) = V_g(\bar{\boldsymbol{q}}(\boldsymbol{\theta})) + V_k(\boldsymbol{\theta} - \bar{\boldsymbol{q}}(\boldsymbol{\theta})) . \tag{43}$$

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